Comparison of Homogenization Schemes to Periodic and Random Simulations

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Assessment of the efficiency of the Interaction Direct Derivative homogenization scheme by comparison to Finite Element Simulations.

Average properties

At a macroscopic scale, for a porous medium with one pore family loaded with pressures p, the constitutive law writes [1]:

$$\begin{cases} \underline{\underline{\Sigma}} = \mathbb{C}^{hom} : \underline{\underline{E}} - p\underline{\underline{B}} \\ \phi - f = \underline{\underline{B}} : \underline{\underline{E}} + pM \end{cases}$$

Where we call $\mathbb{C}^{hom}$ the homogenized stiffness tensor, $\underline{\underline{B}}$ the Biot coefficient, and M the Biot modulus (inverse of the usual Biot modulus N).

Simulations using `FreeFem++`

We create a volume with pores in a 2d plane strain model, and apply:

- external loadings
- pressure in the pores

using periodic B.C., to determine the poroelastic constants by measurement of strain and stress averages.

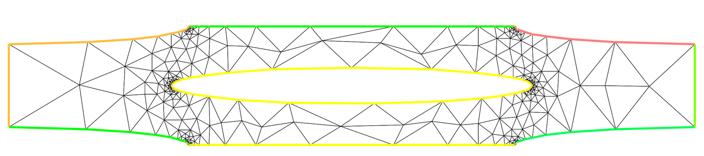


Fig. 1: Elliptical inclusions on an elliptical grid, periodic simulation

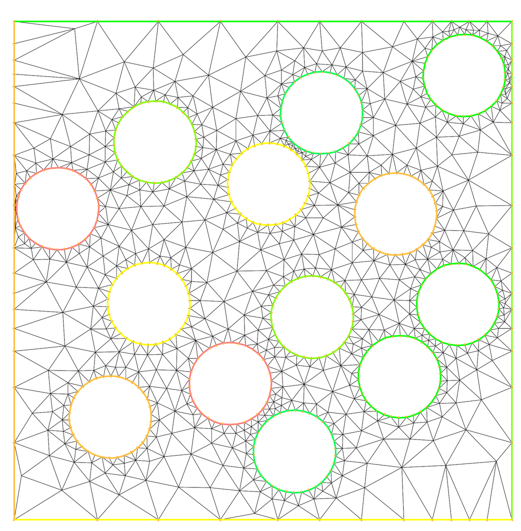
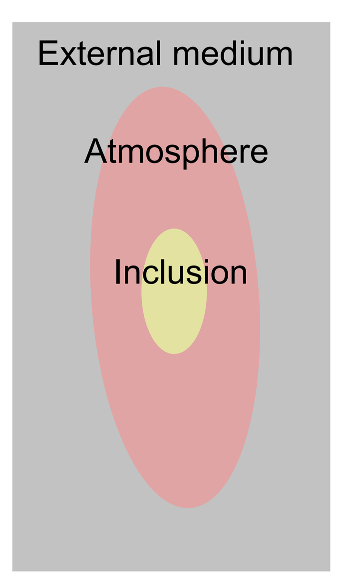


Fig. 2: Circular inclusions in a random simulation

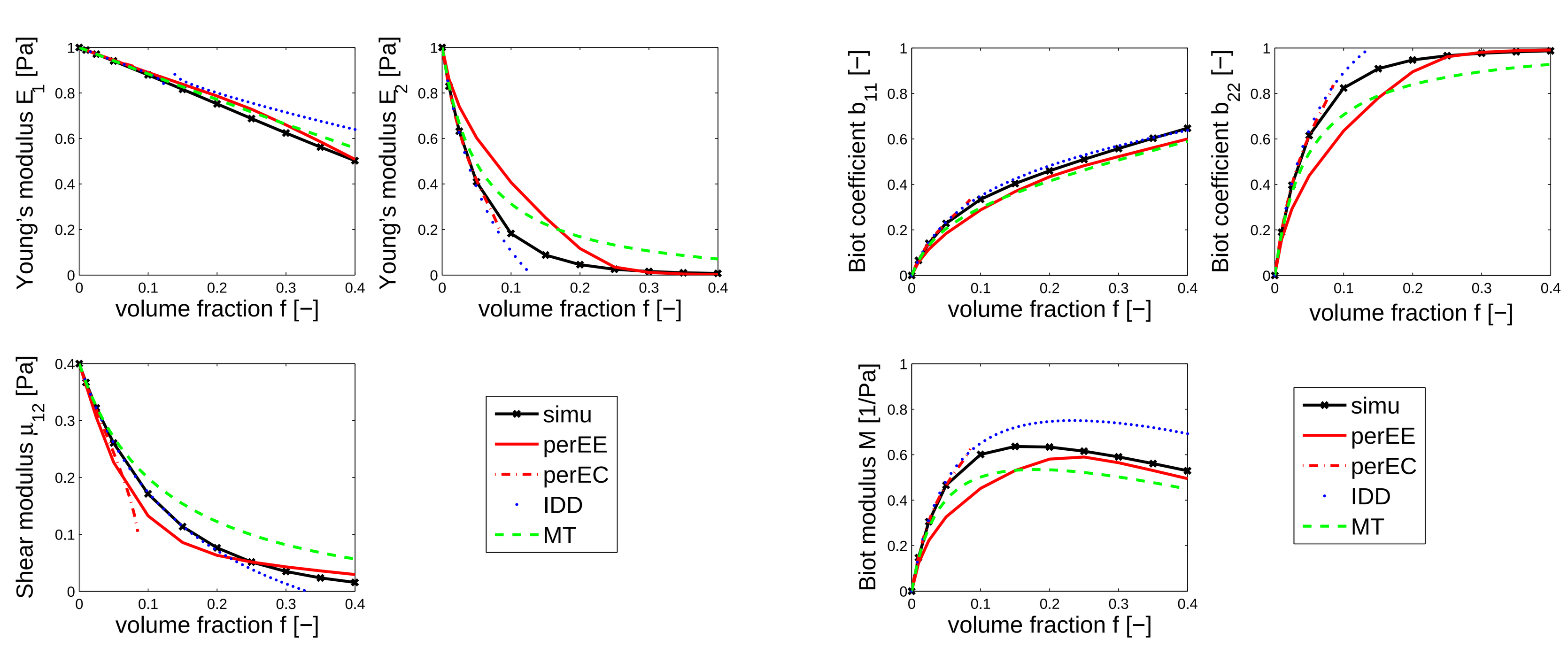
Estimates

- Mori-Tanaka: **MT** (Inclusion embedded in the matrix, averages computed on a domain of the same shape as the inclusion, respecting volume fractions)
- Interaction-Direct-Derivative: **IDD** (Convenient simplification of the generalized self-consistent scheme, in which the inclusion is embedded in the matrix atmosphere, which is embedded in the average medium [2])

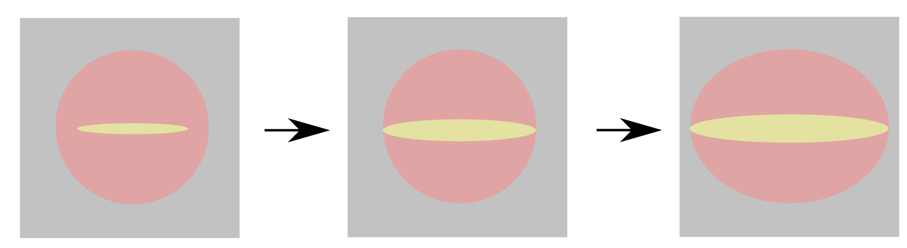


Aligned elliptical pores, aspect ratio 0.1

We compare random simulations **simu**, periodic simulation with isotropic cell **perEC** and elliptic cell **perEE**, to the **IDD** scheme with circular atmosphere and the **MT** scheme. The **IDD** is failing because some coefficient reach their bound (0 or 1) far too early. **MT** does not show this problem. **IDD** needs to be used more carefully. **perEC** is accurate but cannot reach high volume fractions, **perEE** gives unsatisfactory results at intermediate volume fractions.



We modify the IDD scheme to improve the results according to a simple geometrical rule and an optimization procedure. The aspect ration of the atmosphere $\frac{b^d}{a^d}$ needs to change from 1 to that of the inclusion 0.1 when the volume fraction f increases.



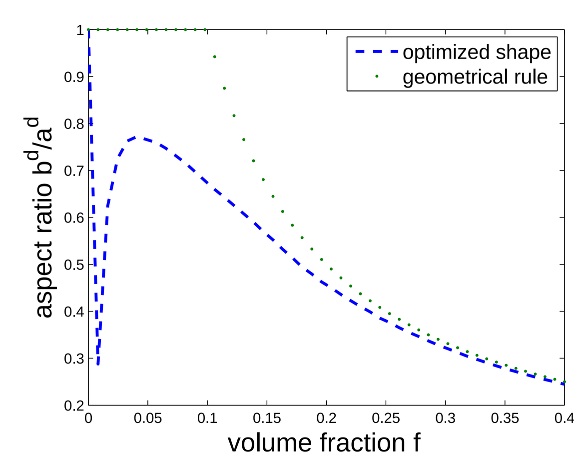


Fig. 3: Two possibilities for the evolution of the aspect ratio of the atmosphere

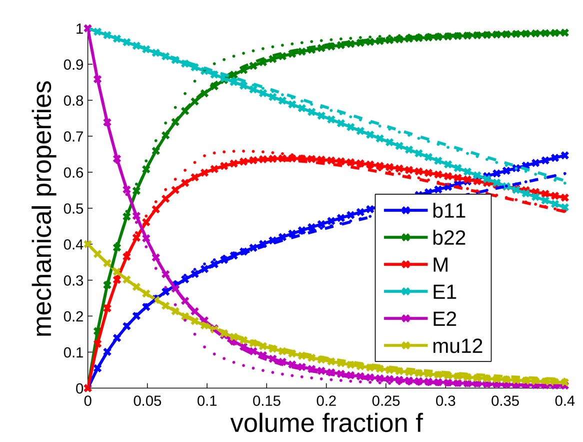
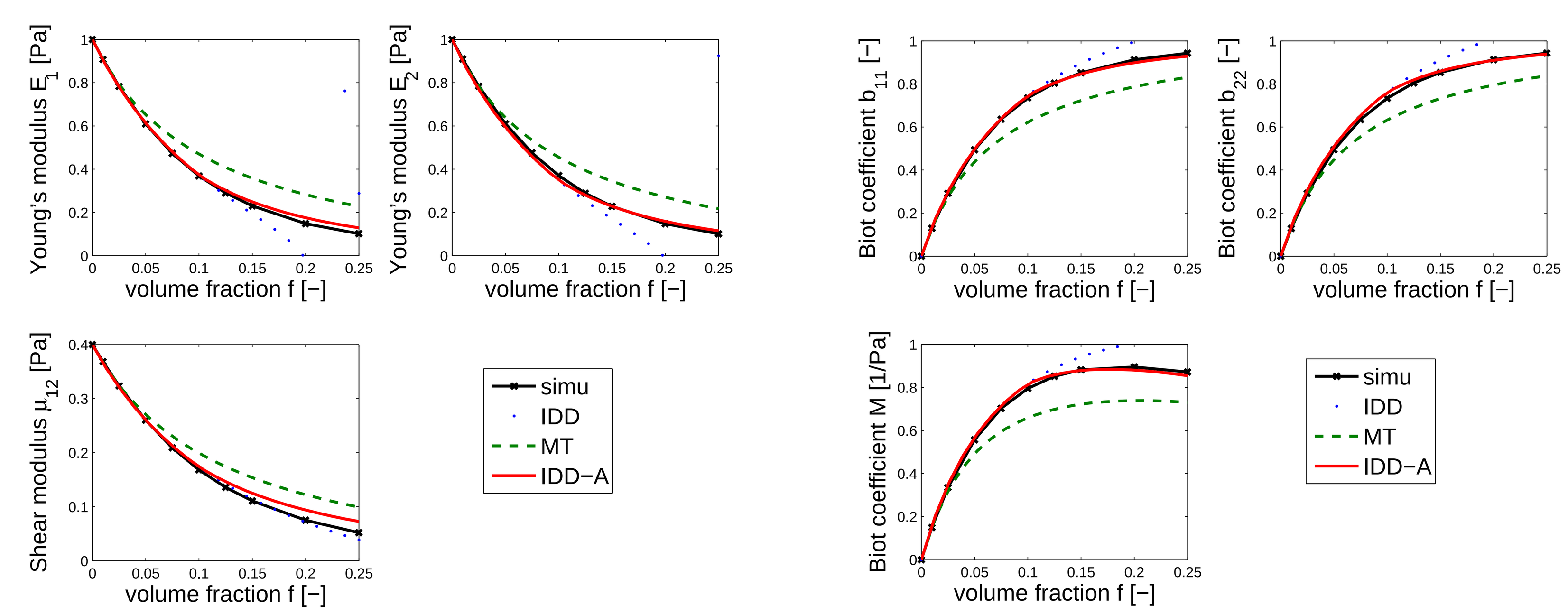


Fig. 4: Efficiency of this modification. (×): simulations, (•): geometrical rule, (−−): optimized shape

The geometrical rule is less satisfactory than the optimized shape, but is simpler. We call the **IDD** estimate built by this modification **IDD-A**. It is not new [3].

Isotropically oriented elliptical pores, aspect ratio 0.1

Finally we compare three estimates to simulations in the case of randomly oriented pores. The results obtained with **IDD-A** are very satisfactory.



Conclusion

The IDD scheme, when used with adapted shapes for the atmospheres, gives good results to predict the homogenized properties of crack-like pores, whether aligned or isotropically oriented.

Acknowledgements

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References

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